

## Teaching Potentially Mathematically Gifted

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### Abstract:

*The country might have attained Independence from foreign rule, but most students are yet to release themselves from 'home rule'. Even apples are graded and packed in different baskets, but the "human apples"- bright, mediocre and dull- are packed in the same classroom! We have hindered the blooming of many gifted and talented who could have spread their fragrance in this world. This reference is particularly for those who gave mathematical principles as Pythagoras theorem, probability, Euclidian geometry, Archimedes principle, coordinate system etc., they have given shape to the world that we see today. Their major contributions have reasoned our survival. How were they different from the normal children? How were their hard wired brains different from others? Researchers have found that even few months old babies notice the differences in numerical quantities (Berger, Tzur& Posner, 2006). Some eight months old infants can distinguish an individual object from collection (Wang& Wynn, 2000). These babies have sophisticated grasp of quantity, what scientist called numerocity (Sausa, 2008), these being early signs of arithmetical talent. How is their processing of information different from other babies? Keeping this question in view the present paper focuses on neuroscientific findings on functioning of brain, identifying mathematically gifted and determining their curriculum to nurture them for the sake of their own happiness and for society as well.*

**Key words.:** Mathematically Gifted, Neuroscience, Identification, Instructional Strategies

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## I. INTRODUCTION

Thirty-year old Ambala born identical twins, Abhay and Ajay Saini hit upon a novel scheme of Population Control'. Both are young engineers. They identified 35 major problems facing the country and fed them into computer. Right from the top as the first priority came- Population. At the bottom of the list Coin Shortage. So, they worked out a fresh scheme of "population control" based entirely on incentives. A brief is as follows:

A couple who agreed to sterilization after they have one daughter will be entitled to a government bond of Rs. 1.25 lakhs encashable after 21 years. Those who do so after the birth of a son, Rs 80 000 encashable+ after 25 years. Those sterilized after two daughters, Rs. 90 000 encashable after 21 years; with two sons, Rs 30 000 encashable after 25 years, and those with son and daughter, Rs. 60000 encashable in case of the daughter after 21 years, 25 years in case of son. This scheme if put in operation will initially cost nothing except the cost of printing certificates and be anti- inflationary as the money remain available to the state for over two decades!

Another excellent figure is Torence Tao. He taught himself arithmetic at age 2, and at age 12 he was youngest ever gold medalist in International Mathematical Olympiad. Earning his doctorate in mathematics at the age of 20, four year later at 24 Tao became the youngest full professor in university of California. Today, after a decade of work in mathematics, he has 140 papers to his credit. He is an excellent example of combining academic ability with creative productivity. He worked with pure mathematics and created a new field of mathematics called compressive sampling. He also redesigned the present day camera that would require only a fraction of data to begin with.

Its Tao, Abhay or Ajay, they have the potential to make this world a beautiful place to live in by utilizing their creativity, and brilliance. Sally Herld Reis (1993) said "the planet's survival depends on how successfully the potential of these promising gifted students are realized". So the demand of time is that the discoverers, researchers, neuropsychologists, educators co-ordinate themselves to design curriculum to water the seed of potential talents in them.

## II. BRAIN PROCESSING IN MATHEMATICAL THINKING

How does neurons in human brain process mathematical operations? Is it dependent on language or visual representation? What kind of functions determines an individual's mathematical competence? Such questions have been the focus of scientific investigations for decades. Some researches gave importance to language while others laid emphasis on symbols. Now the neuroscience techniques are producing some fascinating aspects about cerebral activity during various types of mathematical operations.

FMRI imaging studies over years have focused on the area of brain where simple arithmetic functioning as addition and multiplication are processed. Case studies by HittmairDelazer, Semenza& Denis, 1994 found that patients whose left parietal lobes were damaged, had difficulty in arithmetical calculation. It was also reported that left parietal lobe of Albert Einstein's was 15 percent larger than the normal. This extensive development of parietal lobe probably occurred early in Einstein's life, when he was giving signs of his dexterity in mathematical and spatial abilities (Witelson, Kigar & Harvey, 1999). However it is still unanswered that

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larger parietal lobe was only due to Einstein's in depth work in mathematical operation.

With development in sophisticated technique of neuroscience, supporting evidences have been found, that the more complicated the mathematical operation, the more areas in brain are activated. Numerical calculations requiring retrieval of arithmetical concepts activates the prefrontal cortex as well as the left parietal area ( Ravizza, Anderson, & Carter, 2008).

Curiosity arises here what exactly takes place in different areas of brain while carrying out numerical processing. Studies have found that different arithmetical operation as multiplication, subtraction, exact and approximate addition activates different areas of brain. (Chocon, Cohen, Moarte, & Dehaene, 1999). During multiplication left parietal lobe was more strongly activated. This is due to Broca's and Wernicke's area which is responsible for language processing in this lobe. Subtraction showed activation of both left and right parietal lobes, as it involves numerical calculation as well as verbal naming of quantity involved. In an another study, brain activity was traced for subject performing exact and arithmetic calculations (Dehaene, Spelke, Pinel, Stanescu & Tsivkin, 1999).

In a task requiring exact addition, subjects were shown two close numbers on cards (e.g.  $5 + 4 = ?$ ) and were asked to identify answers on followed card (7 or 9?). This exercise was repeated for approximate calculation (3 or 8?). The results were surprising. Subjects took lesser time for exact calculations. In the other round, problems were presented in other language than native language of subjects. It is amazing to conclude that subjects took relatively more time for exact calculations. These findings show that exact calculation is language dependent. But no difference in time was noticed for approximate calculation, indicating that these operations were independent of language. The fMRI results for above study revealed that exact calculation activated left frontal lobe which is associated with language tasks. In contrast approximation tasks activated the left and right parietal and portions of the right occipital lobes showing that these tasks are associated with visual spatial operation.

### **Implication for Studying Mathematics**

These micro studies suggest that students with stronger neural connections between both numerical processing and language center are likely to be more proficient at mathematics than those excelling in only one area. This study has strong implications for the teachers, as it promotes the need to emphasize on verbal language as well as numerical skill in order to facilitate students' mastery in mathematics.

## **III. IDENTIFYING THE MATHEMATICALLY GIFTED**

Mathematically promising students are not always easy to spot. They may doubt their abilities and may prefer to remain in background. This is particularly true for girls. As a result their competence may not be identified at all.

### **Some Attributes of Mathematical Giftedness**

Students with high mathematical ability are able to-

- Learn and understand ideas quickly
- Display multiple strategies for solving problems
- Engage other students in their activities
- They make convincing arguments about their views and try to enroll others into their activities
- Sustain their concentration and show great tenacity in pursuing solutions
- Quickly recognize similarities, differences and patterns
- Look at problem more analytically and holistically

### **Some myths about mathematically gifted students:**

Some myths those are associated with mathematically talented are-

- Mathematically talented students are outstanding at computation.
- Results from school grade level evaluation and standardized test are sufficient for identifying mathematically talented.
- Such students demonstrate mastery of a topic by earning 100% on tests.
- Students who are accelerated<sup>2</sup> need not cover each section of the text.
- They cannot be identified until high school.

The fact that these myths are wide-spread indicates and demands the development and adoption of appropriate curriculum for educating mathematically gifted.

### **Mathematical Giftedness and Mathematical Creativity**

Usiskin (2000), tried to demarcate the mathematically creative from the mathematically gifted. He used eight tiered hierarchy ranging from level 0 to level 7. Level 0 represents adults who know very little mathematics. Level 1 represents adults whose mathematical knowledge is comparable to those of students in Grade 6 through to Grade 9. A very large proportion of general population would fall in the first two levels. Level 3, are those who are capable of taking mathematics as major subjects and become mathematics teachers in Secondary Schools. Level 3 are the 'terrific' students who score 750-800 range on SATs and have the potential to do beginning graduate level work in mathematics. At level 4 we have the 'exceptional students who excel in mathematics olympiad and competitive examinations. Because of their talents they can converse intelligently with Mathematicians about mathematics and can construct mathematical proofs.

Level 5 represents productive Mathematicians, who have completed mathematical sciences and are capable of publishing in world class journals. Level 6 is the territory of exceptional mathematicians who will move their field forward and who will be found in any history of the field in which they work. Level 7 is the exclusive territory of exemplary genius like Leonard Euler, Karl Friedrich Gauss, Ramanujan etc.

<sup>2</sup>by acceleration we mean those students who are completely comfortable with the subject matter and their teachers also do not need to repeat the topic.

It is important here to note that gifted mathematicians are at level 5 through 7 whereas creative mathematicians are found at level 6 and level 7. Therefore, in this model creativity implies mathematical giftedness, but the reverse is not necessarily true.

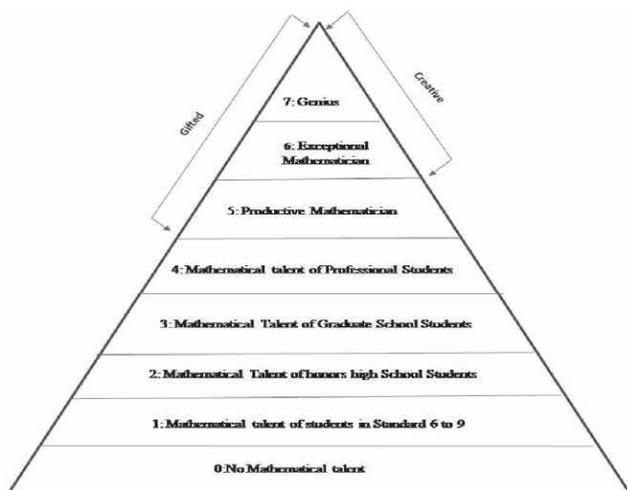


Figure 1: This model of Usiskin's hierarchy classifies mathematical talent into eight levels of mathematicians in level 5 through 7 are gifted, but only those in 6 and 7 are creative.

Source: Usiskin 2000

Sriraman(2005), further differentiates mathematical giftedness and mathematical creativity-

Mathematical giftedness means being able to quickly understand known Mathematical concepts and perform Mathematical operations at a level well beyond what is typical for the individual's age and schooling.

Mathematical creativity includes the characteristics of giftedness and ability to produce original work that significantly extends the body of knowledge, or poses new questions for other Mathematicians.

Application: The earlier the school personally identify the mathematically gifted and creative students, the better plan can be chalked out for them, because early years are important for developing the cerebral areas and establishing the neural networks that perform and manipulate mathematical operations.

#### IV. TEACHING MATHEMATICALLY GIFTED: SOME PEDAGOGICAL TECHNIQUES AND STRATEGIES

Here are some suggestions for differently instructing for Mathematically gifted inside and outside the classrooms (Johnson, 2000; Shoplik, 2006).

##### Assessment related strategies

- Give pre-assessment to determine which student already know the material. Work with these students who do not know the basis and allow the gifted students to complete more complex learning tasks.
- Whole grade acceleration can also be done if the child is talented in all the subject areas. Advantages include being exposed to more challenging material. The potential

disadvantage to this acceleration is that the pace of the new class might be too slow for each learners.

##### Curriculum Materials

- Select the study materials that offer enriched opportunities as most texts are written for average students and are not appropriate for the gifted. Hence multiple research journals, research reports, use of technology to provide the gifted, opportunity to reach the depth and breadth of learning matter. Use of spreadsheet, databases, graphic, scientific calculators can lead to powerful data analysis. Besides, providing access to internet, provide them vast sources of information ideally not found in the text book.
- Curriculum compacting can be another way to provide mathematically gifted the opportunity to study enriched topics. An effective way of compacting is to answer two questions (1) What does the students know? (2) What does the students need to learn? The question can be addressed by teachers by using standardized or teacher made tests.

##### Instructional Techniques

- Instructional techniques require flexibility in pacing. Some students may be mastering basic skills while other would be working on advanced topic.
- Use inquiry based discovery learning that emphasize open-ended problem. Explore enrichment topics- Some topics in elementary level of maths may include-probability, statistics, estimation, mental arithmetic, spatial visualization, algebra, geometry etc.
- Offer level courses in statistics calculus and computer science. Student should be encouraged to- take class at local college if they have exhausted all topics at secondary level.
- Permit the students to work on independent study projects to supplement regular Mathematics curriculum.
- Offer opportunities to participate in contests, such as Mathematical Olympiad and eCybermission. Give feedback to students on their performances.

##### Selecting Teaching Strategies for Mathematically Gifted

Teaching Strategies in Mathematics for gifted students should aim to-

- Develop higher level thinking by challenging students to observe, compare, hypothesize, criticize, classify, interpret and summarize.
- Students should not be expected to work in undirected and unsupported way for extended period of time.
- Strategies should have clear objectives and be designed to increase the student's ability to analyze and solve problems, to stimulate creativity and to encourage initiative and self-direction.
- Offer opportunities for extended research in areas of student's interest
- An example of enrichment opportunity for the gifted is here-
  - o Encouraging the students to research the origin of Mathematical theories/ biographies of Mathematician (e.g. Pythagoras, Euclid, Sophie, Germain, etc)

- Challenge individual students or a small group of talented Mathematically gifted to find many varied ways to arrive at the same solution (e.g. Using repeated addition vs. multiplication or making a pictorial depiction vs. using computation only)
- Using deductive reasoning for problem solving- such as the areas of rectangle, triangle and parallelogram are all related and can be used to find one another.
- An individualized strategy may include the use of Entry and Exit Card. An entry card is a problem question or a task given at the beginning of the class period, while an exit card is a problem or a task given at the end of class period. Use of several cards in a single class can help a teacher to tailor his teaching according to the need of students. This may help in differentiating the poor, average and students who have records of excellence in mathematics. The excellent performers in the class deserve rigorous stimulation and if given work below their readiness they are likely to become frustrated. Since the entry and exit card strategy helps us in separating the competent from the others, teachers can make sure that these students are not frustrated with less challenging tasks.

What question do you have about last night homework? Which problems challenged you most/least? Why?

Example of Entry card

Draw a graph and label the 'x' and 'y' axes. Graph a line with end points (3, 5) (5,2). Graph a line with end points (-3,-5) and (7,2)

Example of Exit card

### V. TALENT SEARCHES PROGRAM FOR MATHEMATICALLY GIFTED

Talent Searches are valuable opportunities for meeting the needs of mathematically gifted elementary and secondary students. Such programmes not only help teachers in identification of the Mathematically gifted but also in guiding and designing educational experience appropriate to the students ability level.

#### The Testing Requirement

Students who score at or above the 95<sup>th</sup> percentile on the composite or Math Total, Vocabulary, Reading, or Science Subtest at the national level (as Math Olympiad) are recommended for further testing. A superior level test can be administered next, usually two to five grade level above the grade of the student. Example of such test are SAT or EXPLORE Test. Indian adaption of such tests can be more fruitful.

#### Interpreting Test Results

Let's take the case of two hypothetical 3<sup>rd</sup> grade students, Student A and Student B, both scored in the 99<sup>th</sup> percentile in the 3<sup>rd</sup> grade test. However on 8<sup>th</sup> grade test, the result showed a very different picture. Student A scored at the 26<sup>th</sup> percentile and student B at the 96<sup>th</sup> percentile. Both these students needed different challenging activities suited to them. Student A need more enrichment in mathematics, participation in contests, group work with students of similar

aptitude in mathematics and curriculum compacting (perhaps, two years of mathematics in one). Student B need all above what student A needed, in addition needed individually paced instruction as well as grade skipping and taking college classes early.

**Table 1.1** shows some guidelines for developing the educational plan for students who achieve different scores in the higher level tests (Rotigel&Lupkowski-Shoplik, 1999).

Tests and Scores	EXPLORE- Mathematics Scale score 1-13 (taken in 4 <sup>th</sup> grade) OR SAT- Mathematics Score of 200-500 (taken in 7 <sup>th</sup> grade)	EXPLORE- Mathematics Scale score 14-20 (taken in 4 <sup>th</sup> grade) OR SAT- Mathematics Score of 510-630 ( taken in 7 <sup>th</sup> grade)	EXPLORE- Mathematics Scale score 21-25 (taken in 4 <sup>th</sup> grade) OR SAT - Mathematics Score of 640-800 (taken in 7 <sup>th</sup> grade)
Components of the Plan	In – school enrichment; participation in competitions and contest  Summer programmes for enrichment	Curriculum compacting ( taking 2 years of mathematics in one year)  Summer programmes of fast- paced classes in mathematics	An individualized program of study based on diagnostic testing in mathematics  Consider grade skipping , early admission to high school, and taking college classes early  Mentorships for advanced study in mathematics
	Algebra I in 7 <sup>th</sup> grade; AP calculus in 11 <sup>th</sup> grade; College-level mathematics courses in 12 <sup>th</sup> grade	Algebra I in 6 <sup>th</sup> grade; AP calculus in 10 <sup>th</sup> grade; College –level mathematics courses in 11 <sup>th</sup> and 12 <sup>th</sup> grades	

### VI. CONCLUSION

Neuroscientific findings support the idea that potentially mathematically gifted students require practice of mathematical operations for extensive period of time. Usiskin (2000) further differentiate mathematical giftedness from creativity. Whereas giftedness is related with faster pace of learning, creativity is taking the field of mathematics further by creating new knowledge. As this field of creativity is full of uncertainty, it requires familiarizing students with history and method of mathematics developed over centuries through extensive study over years, individualized instruction, early college entry etc. All these would develop perseverance in students which will encourage them for hard work and development of insight for problem solving in newer way. Thus policy makers and curriculum designers have great responsibility of early identification and designing appropriate curriculum at school level for nurturing these potentially gifted for solving emerging worldly problems.

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